Teaching notes, The Intertemporal Approach to the Current Account and Other Issues

Signe Krogstrup
Graduate Institute of International Studies, Geneva

March 23, 2005

Abstract

These teaching notes are meant as part of an introduction to the course on "Topics in International Financial and Monetary Policy". They introduce the students to the intertemporal approach to the current account using a very simple two period small economy framework. Issues of current account sustainability and Ricardian equivalance are also touched on. The presentation follows and borrow from the …rst few chapters of Obstfeld and Rogo¤'s "Foundations for International Macroeconomics", 1996, but blends in other treatments of the topics when appropriate.

1 The Intertemporal Approach to the Current Account

The intertemporal approach to the current account stresses the real and long term arguments for why countries might want to temporarily - or even rather persistently - run current account de…cits or surpluses, and hence, that it is not always optimal to have a current account balance. Just as persons might want to borrow more in their early life, for example to acquire education which will enhance that persons income later on in life, so might countries want to borrow more internationally in a phase of economic development, in order to built up production capacity which may enhance growth and hence income later on.

The intertemporal approach does not help explaining short term swings in current accounts, and should hence be combined with models of short term deviations of current accounts.

When looking at history, it is generally easy to identify whether a short term or a long term argument should be used for explaining a certain current account development (i.e. was a given current account movement a short term increase or fall in the balance, with a correction a few years later, or was it a persistent and relatively smooth movement spanning maybe decades before it reversed?). But when it comes to evaluating a given contemporary current account balance, whether it is a manifestation of long term forces or a short term deviation from trend is never straightforward to ascertain. Hence, it is useful to have a long term as well as a short term interpretation of whether a given current account is sustainable.

1.1 The Building Block of the Model

1.1.1 Simple approach, two period small country model:

This model is drawn from O&R, Chapter 1.1 and 1.2. We start directly with a model which includes investment.

We use the most simple setup possible: The model comprises a small open economy within the world economy, which exists for another two periods starting with period t (we hence have periods t and t + 1 to characterize). International capital mobility is perfect, and assets are perfect substitutes internationally. There is no uncertainty and hence no risk in the model. Agents therefore have perfect foresight.

1.1.2 Production

There is only one homogenous good which is produced all over the world. This good is equally used for consumption and investment. Production of the good requires capital. The aggregate production function for period t production of the small country in question is given by

$$Y_t = F(K_t) \tag{1}$$

where the marginal productivity of capital is decreasing in capital, but production is strictly increasing in capital:

$$F'(K_t) > 0,\ F''(K_t) < 0 \text{ and } F(0) = 0 \tag{2}$$

The level of capital of any given period is given by the historical level of capital plus investment in the previous period:

$$K_{t+1} = K_t + I_t \tag{3}$$

The level of capital in period t is thus given by history and decisions made in the previous period. Hence, K_t can be treated as pre-determined. K_{t+1} is, however, not yet determined in period t, but will depend on investments made in period t. Note that investments can be negative, and will be negative in the last period, t + 1, since there is no reason to have a positive level of productive capital going forward when the economy seizes to exist after period t + 1:

$$I_{t+1} = -K_{t+1} \tag{4}$$

1.1.3 Government

The government levies lump sum taxes on each household of an amount τ_t and τ_{t+1} in the two periods respectively, and purchases public goods, G_t and G_{t+1} in each period. For simplicity, we treat spending in the two periods as exogenous, and assume that that the government can finance a potential period budget deficit (surplus) by selling (buying) treasuries, B_t^G, in the international capital market at the world interest rate. We also assume that the government sets tax rates which fulfill the intertemporal government budget constraint:

$$G_t + \frac{G_{t+1}}{1+r} = \tau_t + \frac{\tau_{t+1}}{1+r} \tag{5}$$

We will not get into how the government chooses its tax rates here.

1.1.4 Households

The country is inhabited by an infinity of identical households. In order to skip a lot of computational complications without losing any insights, we normalize the total number of households to one[1]. We refer to this household as the representative household, or just the household, in the following.

Utility In period t, the representative household derives utility from consumption in period t and from the expected consumption in period t + 1 according to the additively separable utility function[2]:

$$U_t = u(C_t) + \beta u(C_{t+1}) \tag{6}$$

where $\beta \in]0;1[$ is the time preference factor. The higher β is, the less impatient are households. The usual assumptions regarding the period utility function, $u(\cdot)$ apply:

$$u'(\cdot) > 0\ ,\ u''(\cdot) < 0 \text{ and } \lim_{C \to 0} u'(C) = \infty$$

The Household's Budget Constraint and International Borrowing The representative household has disposable income from ownership of production net of taxes, and spends this income on consumption, taxes and savings as described in the following:

[1] A more elaborate explantion of this simplication is that we assume an infinity of identical households located on the interval from zero to one. That they are identical means that their consumption, investment and borrowing choices are always the same, and are equal to per capita variables (i.e. individual i's consumption level in period t is equal to per capita consumption in period t as well). That the households are distributed along the interval from zero to one, such that there is an infinity of infinitely small households and their total number is one, in turn allows us to treat per capita variables as national aggregate variables (i.e. household i's consumption level in period t is equal to the per capita consumption level in period t, which in turn in equal to aggregate consumption in period t).

[2] Note that expected consumption in period t + 1 is equal to actual consumption in period t + 1 since there is no uncertainty in the model

Income: The representative household owns all installed capital, K_t, and is paid the marginal return to capital per unit of capital in each period, $F'(K_t)K_t$. The household also owns the shares of the production process and gets the period profits paid out as dividends each period. Per period profits, X_t, amount to $X_t = F(K_t) - F'(K_t)K_t$. Adding these two sources of income sums to aggregate income, $X_t = Y_t = F(K_t)$.

Borrowing and lending in international capital markets Households have free access to borrowing and lending in international capital markets at the international interest rate, r. Since the household spends resources consuming and investing in period t, the net amount of borrowing of the small economy in question is equal to

$$B_t^P = C_t + I_t - (Y_t - \tau_t). \tag{7}$$

where B^P is private borrowing. Since we assume that there is no uncertainty and that households always honor obligations, the net borrowing of period $t+1$ becomes

$$B_{t+1}^P = C_{t+1} + I_{t+1} + (1+r)B_t^P - (Y_{t+1} - \tau_{t+1}). \tag{8}$$

Note that access to international capital markets allows the household to smooth consumption over time in cases where income is volatile (or just differs across time), which will increase utility due to the assumption of decreasing marginal utility of consumption.

The household intertemporal budget constraint: In period $t+1$, the household cannot hold any debt since foreign lenders would not agree to this. They are also not interested in leaving any net savings unused. Hence, $B_{t+1}^P = 0$ for the intertemporal budget constraint to hold. Inserting this in (8) above, and substitution B_t^P out in (7), we get the net present value intertemporal household budget constraint[3]:

$$Y_t - \tau_t + \frac{Y_{t+1} - \tau_{t+1}}{1+r} = C_t + I_t + \frac{C_{t+1} + I_{t+1}}{1+r}. \tag{9}$$

Holding income constant, we see that the trade-off between period t and period $t+1$ consumption in terms of the budget is $-(1+r)$, which is the slope of the budget line in a graph of intertemporal consumption such as the red or green line in Figure 1.

1.1.5 The National Income Identity and the Current Account

We briefly introduce some familiar identities relating to the current account in the present model, for later use. Private savings are defined in the usual way as households' disposable income less consumption (here for period $t+j$, $j = 0, 1$):

$$S_{t+j}^P = Y_{t+j} - \tau_{t+j} - C_{t+j} - rB_{t+j-1}^P \tag{10}$$

[3] We assume equality in the budget constraint since the household will not be interested in leaving any resources behind when the economy stops after period $t+1$.

Note that we assume $B^P_{t-1} = 0$. Public savings in period $t + j$ are equal to government income less government spending:

$$S^G_{t+j} = \tau_{t+j} - G_{t+j} - rB^G_{t+j-1} \tag{11}$$

Total savings in the economy thus sum to

$$\begin{aligned} S^G_{t+j} + S^P_{t+j} &= S_{t+j} = Y_{t+j} - rB_{t+j-1} - rB^G_{t+j-1} - G_{t+j} - C_{t+j} \\ &\Rightarrow S_{t+j} = Y_{t+j} - rB_{t+j-1} - G_{t+j} - C_{t+j} \end{aligned} \tag{12}$$

where B_{t+j} is the total net foreign debt of the country in period $t + j$. Savings add to net wealth each period. Net wealth can be held in the form of domestic capital, K, and in the form of bonds issued in international capital markets, $-B$ (minus since B is defined as debt here). The change in net wealth between period t and $t + 1$ comes from households savings in period t, so that:

$$S_t = K_{t+1} - B_{t+1} - [K_t - B_t]$$

Now, substitute the capital accumulation equation, (3), into the above to obtain

$$\begin{aligned} S_t &= K_t + I_t - B_{t+1} - [K_t - B_t] \\ &\Rightarrow S_t - I_t = -(B_{t+1} - B_t) \end{aligned} \tag{13}$$

Note also that substituting the period t version of (12) into $S_t - I_t$, we get:

$$S_t - I_t = Y_t - rB_{t-1} - G_t - C_t - I_t = NX_t - rB_{t-1} = CA_t \tag{14}$$

Hence, we have

$$S_t - I_t = -(B_{t+1} - B_t) = CA_t \tag{15}$$

Equation (15) shows that the current account is equal to excess of savings over what is used for investments in the Home country, and that this excess is placed in international capital markets. In real terms, excess domestic savings over domestic investment will be lend to foreigners in the shape of an export of domestically produced goods.

1.2 Optimization

The household maximizes utility, (6), with respect to consumption and investment in the two periods, and subject to the budget constraint (9), the production function, (1), the capital accumulation function (3) and the final period investment constraint, (4). Substitute the production function and C_{t+1} from the budget constraint into the utility function to obtain:

$$\begin{aligned} U^i_t = {} & u(C_t) + \\ & \beta u((1+r)(F(K_t) - \tau_t) + F(K_{t+1}) - \tau_{t+1} - (1+r)(C_t + I_t) - I_{t+1}). \end{aligned}$$

Then, substitute the capital accumulation function (3) and the final period investment constraint, (4), into the above to obtain:

$$\begin{aligned} U_t^i &= u(C_t) + \\ &\quad \beta u((1+r)(F(K_t) - \tau_t) + F(K_t + I_t) - \tau_{t+1} - (1+r)(C_t + I_t) + K_t + I_t). \end{aligned}$$

Now differentiate with respect to C_t and set equal to zero to get the first of the two first order conditions (FOC 1):

$$\begin{aligned} 0 &= u'(C_t) + \beta u'(C_{t+1})(-(1+r)) \\ \Rightarrow &\quad \frac{\beta u'(C_{t+1})}{u'(C_t)} = \frac{1}{1+r}. \end{aligned} \tag{16}$$

This is the famous Euler equation, giving the optimal path of consumption in time. The intuition behind this condition is the following. Consider an decrease present consumption by dC; which will be transferred to the future for future consumption. In the present, this reduces utility by $u'(C_t)dC$. In the next period, there will be $dC \cdot (1+r)$ funds available for consumption. This increase in consumption will increase utility by $\beta u'(C_{t+1})dC \cdot (1+r)$. As long as the change in utility in the two periods are not the same, it will be optimal for the household to transfer funds from the period with lower utility to the period with higher utility change. Since transferring consumption toward the period with higher chance in utility will reduce the marginal utility of consumption in that period, and vise versa, it must be the case that the utility change in the two periods due to a transfer of funds is the same in equilibrium. Thus, $\beta u'(C_{t+1})dC \cdot (1+r) = u'(C_t)dC$. This reduces to (16) above. Graphically, condition (16) holds in the point where intertemporal indifference curves, with slope $-\frac{u'(C_t)}{\beta u'(C_{t+1})}$ are tangent to the intertemporal budget constraint, with slope $-(1+r)$. Fig 1, point A or C for an example.

The second first order condition is found in the same way by differentiating with respect to I_t, which yields the well known optimal investment condition (FOC 2):

$$\begin{aligned} 0 &= \beta u'(C_{t+1})(F'(K_{t+1}) - (1+r) + 1) \\ \Rightarrow &\quad F'(K_t + I_t) = r \end{aligned} \tag{17}$$

Condition (17) shows that the condition for optimal investment does not depend on time preferences or other characteristics of the consumer's preferences, but only on whether the return to investment exceeds or falls short of the world interest rate. The intuition is simple. The household can borrow freely at the world interest rate. As long as returns from investment are higher than this interest rate, it is possible to borrow internationally and place the loan in investment, without changing any other decisions. The next period, the investment pays off more than the cost of the loan. Hence, all else equal, the household has a higher present value of disposable income, which can be transferred between present and future consumption at the international interest rate. On the other hand, if the interest rate is higher than the return to investment, the opposite holds, and it would make sense for the household to disinvest and place the money in international bonds, thereby increasing the return to savings. In optimum, however, condition (17) holds, since changes in investment change the return to investment in an equilibrating way.

We now have two …rst order conditions to solve for C_t and I_t, while C_{t+1} and I_{t+1} are determined residually using the budget constraint, (9) and the …nal period investment constraint, (4) respectively.

1.3 The Production and Consumption Possibilities Frontier and Intertemporal Trade

We will now look at the optimal allocation of funds between consumption, investment and international bonds graphically. The graphical illustration is familiar from trade theory, and the intuition is nearly identical. Here, the relative price of interest is that of time, r, and the autarky relative price will di¤er among countries according to their autarky return to investment (and many other factors, such as demographics, which are outside the present simple modeling setup). The …rst graph that we need to plot is the production possibilities frontier.

1.3.1 The intertemporal PPF

The PPF shows the technological possibilities available for transforming available period t private consumption goods into available period t+1 private consumption goods through period t investment. To derive an expression for it, see that available resources for investment in an autarky situation are equal to:

$$I_t = F(K_t) - G_t - C_t$$

Moreover, period t + 1 production of private consumption goods and what is available through …nal period disinvestment is

$$C_{t+1} = F(K_t + I_t) + K_t + I_t - G_{t+1}$$

Plugging the former into the latter, we obtain the PPF:

$$C_{t+1} = F(K_t + F(K_t) - G_t - C_t) + K_t + F(K_t) - G_t - C_t - G_{t+1} \qquad (18)$$

Expression (18) gives the downward sloping concave curve in Fig 1. The two corners of the area, the intercept with the period t + 1 consumption axis, and the intercept with the period t consumption axis, are given by the extreme situations where a), all production in period t is invested in period t + 1 production, and b) all production plus capital in period t is consumed, leaving a zero production in period t + 1.

The area is strictly concave due to the diminishing marginal productivity of capital (totally di¤erentiating (18) gives us the expression for the slope: $\frac{dC_{t+1}}{dC_t}$ =

$¡(1 + F^0(K_t + I_t))$. Moving from point b to point a, the more investment is made in period t, the lower is the return to that investment, and the less of a consumption increase it entails in period $t + 1$. Note that in the point where the PPF is tangent to the intertemporal budget constraint, the slopes of the two curves are identical and it therefore must be that $¡(1 + F^0(K_t + I_t) = ¡(1 + r)$. This reduces to the FOC 2, (17), above.

1.3.2 Indi¤erence Curves and the Autarky Situation

Using the utility function, we can plot di¤erent indi¤erence curves in Fig 1. The optimal autarky point is where the highest indi¤erence curve is tangent to the PPF, point A in Fig 1. Drawing the straight line tangent to the indi¤erence curve and the PPF in this point gives us the autarky intertemporal price of period t consumption in terms of period $t + 1$ consumption, the gross interest rate, $1 + r_A$. All optimality conditions are ful…lled in this point - the household's euler equation is ful…lled, since the slope of the indi¤erence curve is equal to the budget line slope, producers are optimally investing, and the resource constraint is ful…lled as the current account and trade balances are zero.

1.3.3 The Open Economy and the World Interest Rate

When the country opens up to intertemporal trade, and thus allows its current account to be temporarily in de…cit or surplus, it can trade intertemporally at the world price of current consumption in terms of future consumption, $1 + r$. The highest iso-budget line reachable at the given world interest rate is drawn in Fig 1 as a tangent to the point B. Now, producers will chose to invest such that B becomes the optimal production point on the PPF. Moreover, the highest attainable indi¤erence curve under the new budget constraint is tangent to the budget line in point C in Fig 1, and this is now the optimal intertemporal allocation of consumption from the point of view of households.

Now, consumption is not equal to production less investment and government spending in each period, since access to international credit allows detaching consumption from production and investment decisions, as we already discussed. This possibility allows consumption smoothing and thus increases the level of attainable utility.

In this example of Fig 1, we have a current account de…cit and international borrowing in period t, and a current account surplus and repayment in the period $t + 1$. The trade account de…cit (note, note the current account de…cit since interest transfers are not included here) in period t is equal to the horizontal di¤erence between points B and C. The vertical di¤erence gives us the trade account surplus in period $t + 1$, which we know is equal to the current account de…cit in period t plus interests.

1.3.4 What happens to the optimal current account when investment productivity is expected to increase?

Assume that the world consists of an infinity of identical countries characterized by the same equations are those of the country we have analyzed above. The only potential difference between the countries is that the production functions can be written as

$$Y_t^i = A_t^i F(K_t^i) \tag{19}$$

where i is an index pertaining to the country, and A is a scalar pertaining to the productivity of country i in period t. The basic production function, $F(\cdot)$, would remain the same across countries and time. This amendment to the model would not change consumption preferences, and the Euler equation (16) would remain the same. The second first order condition - the one for optimal investment in period t - changing to

$$A_{t+1}^i F'(K_t^i + I_t^i) = r \tag{20}$$

and the expression for the intertemporal PPF changing to

$$C_{t+1}^i = A_{t+1}^i F(K_t^i + A_t^i F(K_t^i) - G_t^i - C_t^i) + K_t^i + A_t^i F(K_t^i) - G_t^i - C_t^i - G_{t+1}^i. \tag{21}$$

The slope of the intertemporal PPF would hence change to $\frac{dC_{t+1}^i}{dC_t^i} = -(1 + A_{t+1}^i F'(K_t^i + I_t^i)$, and would hence only be changed by the value of the productivity factor in the second period, A_{t+1}^i.

Assume first that $A_t^i = A_{t+1}^i = A$ for all i. In this case, we have symmetric countries and we would not have any gains from intertemporal trade. Opening up to capital markets would not benefit, nor hurt, anyone. The optimal current accounts in period t and t + 1 would be zero for all countries and the interest rate would be the same as the autarky interest rate.

Now consider country H, for Home, and let A_{t+1}^H increase while keeping all other A_{t+1}^i at the same level as before. This would represent an increase in the expected return to investment in country H while no such investment productivity increase would be expected in the rest of the world. Figure 2 illustrates this example. The PPF shifts upward, so that produced consumption increases in period t+1 while it stays the same in period t since nothing happens to period t productivity. This means that the PPF becomes steeper along the entire curve

The new optimal production point shifts to the left and upward (from point A to point B in Figure 2). What happens is that returns to investment increase while the cost of financing this investment stay put at the world interest rate. Hence, more of domestic production is channeled into investment in period t (which is why the production point in period t, which gives production less investment, shifts to the left in Figure 2), and less goods are available for consumption in period t.

At the same time, the slope of the budget line does not change, but the net present value of income and wealth of the Home country increases. Hence, the households' intertemporal budget line shifts outward, to the red line in Figure

2. As we have drawn the indi¤erence curves in Figure 2, this implies that consumption increases in both periods[4].

To sum up, an increase in expected productivity of investment in one country, which is not mirrored in other countries, implies that it becomes optimal for that country to run a current account de…cit in order to …nance a higher level of investment without cutting in present consumption levels. This allows the country to bene…t from higher levels of income in future while smoothing the e¤ect of this higher level of future income on consumption.

1.4 Conclusions

What is the main message of this simple model of the intertemporal approach to the current account? There are three points to make.

1. **Temporary current account imbalances can be optimal,** as long they are met by opposite current account imbalances in future and net foreign obligations are honored.

2. **Optimality of a current account imbalance due to consumption smoothing and investment …nancing:** When might a current account de…cit be optimal according to this model? When the autarky interest rate in the country in question is higher than the world interest rate. And this happens when the return to investment is higher in the country in question relative to other countries. In this case, borrowing abroad to invest allows an increase in second period production which exceeds the interest cost of this investment. Moreover, borrowing to invest rather than reducing household consumption to investment - thereby smoothing out consumption across time - allows the country to reach a higher level of utility.

3. **Growth implies optimality of current account de…cits, all else equal.** We can generalize and say that according to this theory, a country which grows above average will …nd it optimal to save less (i.e. smooth the higher future expected production into higher current consumption), invest more and therefore run current account de…cits, all else equal. The higher future output will in turn provide for future current account surpluses which will repay incurred net foreign debts.

The traditional idealized example of this has been developing countries, which could use a current account de…cit and capital imports as means of …nancing high levels of investment in growth industries, which would then pay o¤ in terms of growth and allow current account surpluses later on.

Example: The current US current account de…cit in the light of the Intertemporal Approach to the Current Account: Take the example of the US and its current account de…cit, which has been persistent over a long period of time, and is now increasing. In the light of the above model, the US would be a country with a PPF which is relatively tilted toward future production, but with indi¤erence curves relatively tilted toward present consumption -

[4] If we assume homothetic preferences, an increase in income will always lead to an increase in consumption of both goods given prices - here an increase in consumption in both periods, given the interest rate.

just as in Figure 2. Access to international capital markets gives the US access to cheap …nancing for investment without having to sacri…ce current consumption. The model thus tells us that the present US current account de…cit can be understood as the US using international …nancial markets to …nance investments in higher future productivity, and that this higher future productivity will allow the US to easily pay o¤ the associated accumulated foreign debt by running current account surpluses in future.

As we will see later in the course, all the premises of this argument can be strongly disputed. A few examples are that the simple model above does not take into account monetary imbalances and uncertainty.

2 Current Account Sustainability

But …rst, we will talk about the concept of sustainability of the current account. This adjective is being used very often to describe what the present US current account de…cit is not, so it is worth while to take a look at what is actually meant by this.

A very simpli…ed de…nition of current account sustainability, which can be applied to our two period model presented above, is that a given current account de…cit can be repaid in future given the country's production possibilities frontier and accumulated capital and assets.

In the model above, the current account de…cit is always sustainable exactly because sustainability is one of the underlying assumptions: The intertemporal budget constraint is assumed to always be ful…lled and there is perfect foresight, so no unexpected shocks to productivity can change the second period ability to repay debts acquired in the …rst period.

But suppose that A_{t+1} in the above model is not directly observable in period t - a not so unrealistic assumption. Suppose moreover, that many previous years of productivity growth implied that if the previous trend continued, the PPF would look like the blue PPF in Fig 2. Assume also that all agents in the model are risk-neutral (so as to not have to deal with risk premia as well). On the background of this expectation, households, producers and foreign investors would decide on consumption, borrowing and investment given by points B and C, just as before.

But in period t+1, it turns out that productivity was not as high as expected, and that the actual productivity was just equal to A_t, implying that the actual, or ex post, PPF is the one represented by the black line in Figure 2. In this case, consumption, investment and the current account de…cit in period t are sunk, and cannot be changed, and the ex post production point would be at point D in Figure 2. To be able to repay the net foreign debts acquired to …nance the current account de…cit in period t, the private consumption in period t + 1 would have to be reduced to point F in the Figure. This might be such a painful contraction that it is deemed less painful to default on the debts. In this case, the current account de…cit of period t was not sustainable, but this only became known in period t + 1.

In this simple example, we would be able to calculate the likelihood that a current account deficit is not sustainable if we know the threshold fall in period $t + 1$ consumption beyond which we would see a default, and if we know the probability distribution of A_{t+1}.

2.1 The Transversality Condition

Usually, the definition of current account sustainability is a bit more nuanced, notably because the real world does not just consist of two periods, does not face a constant real interest rate, and purely monetary factors play an important role in a world of floating nominal exchange rates. A more nuanced and practical definition could be that sustainability refers to the ability of a country to be able to continue a given current account deficit over the medium to long term without it leading to an accumulation of net foreign debt, which eventually will be so large that there will be doubts as to the capability or willingness of the debtors to honor the debts.

This is the type of definition which is generally referred to when current account sustainability is being discussed. A formalization of this definition could be given within the framework of the above model extended to the multiple period case. Rather than setting up and solving the entire model here, we will just look at national income accounts, budget constraints and balance of payments definitions (the entire model is laid out in Chapter 2.1 of Obstfeld and Rogoff). We simplify relative to the two period model above by assuming away the government and study an endowment economy, where income, Y_t, is given to us exogenously each period. We maintain the assumption of perfect foresight, so as to abstract from risk and uncertainty issues, and we assume a constant interest rate over time, r. And crucially, assume that rather than two periods, we start out by looking at a T period economy.

First, look at the development of debt over time, according to the period budget constraint of the household:

$$B_{t+1} = (1 + r)B_t + C_t - Y_t$$

Divide both sides by $(1 + r)$ and isolate B_t. Write the same equation for another couple of periods:

$$B_t = \frac{B_{t+1}}{(1 + r)} + \frac{Y_t - C_t}{(1 + r)}$$

$$B_{t+1} = \frac{B_{t+2}}{(1 + r)} + \frac{Y_{t+1} - C_{t+1}}{(1 + r)}$$

$$B_{t+2} = \frac{B_{t+3}}{(1 + r)} + \frac{Y_{t+2} - C_{t+2}}{(1 + r)}$$

By iterative substitution until period $t+T$, we get one version of the consolidated budget constraint for households in the economy we are inspecting:

$$
\begin{aligned}
B_t &= \frac{1}{(1+r)}\left[\frac{B_{t+2}}{(1+r)} + \frac{Y_{t+1} - C_{t+1}}{(1+r)}\right] + \frac{Y_t - C_t}{(1+r)} \\
&= \frac{1}{(1+r)}\left[\frac{1}{(1+r)}\left[\frac{B_{t+3}}{(1+r)} + \frac{Y_{t+2} - C_{t+2}}{(1+r)}\right] + \frac{Y_{t+1} - C_{t+1}}{(1+r)}\right] + \frac{Y_t - C_t}{(1+r)} \\
&= \vdots
\end{aligned}
$$

$$
\Rightarrow B_t = \sum_{i=0} \left(\frac{1}{1+r}\right)^{i-1} (Y_{t+i} - C_{t+i}) + \left(\frac{1}{1+r}\right)^{T-1} B_{t+T} \qquad (22)
$$

Now, the terminal condition for this economy to be honoring its intertemporal budget constraint is that there is no debt left in final period, i.e. $\left(\frac{1}{1+r}\right)^{T-1} B_{t+T} = 0$. Take the limit of this expression for $T \to \infty$ to get what we call the **transversality condition** for the economy[5]:

$$
\lim_{T \to \infty} \left(\frac{1}{1+r}\right)^{T-1} B_{t+T} = 0 \qquad (23)
$$

The same thing can also be expressed by imposing the terminal condition in expression (22) and taking the limit as T goes to infinity:

$$
B_t = \sum_{i=0} \left(\frac{1}{1+r}\right)^{i-1} (Y_{t+i} - C_{t+i}) \qquad (24)
$$

(24) simply states that the level of foreign debt in period t has to be matched by the sum of trade account surpluses in all future. This links up with the current account sustainability in that $CA_t = Y_t - C_t - rB_t$ - i.e. the trade balance less net interest transfers abroad. A country always needs to be able to meet its future debt service obligations through an appropriate average path of income less consumption

But how can a country violate the transversality condition, when there is always an infinity of time to correct any current imbalances? To see this, assume that all future income is known and exogenous in period t, and thus that the discounted value of all future income was known and constant, $\sum_{i=0} \left(\frac{1}{1+r}\right)^{i-1} Y_{t+i} = \sum_{i=0} \left(\frac{1}{1+r}\right)^{i-1} \overline{Y}$, while this is not the case for consumption. Assume also that there is a minimum level of consumption, call it $\overline{C}$, under which the country will

[5] The presence of $\lim_{T\to\infty} \left(\frac{1}{1+r}\right)^{T-1}$ in the expression above, which obviously goes to zero as T goes to infinity, shows that in the infinite horizon case, the limit debt level does not need to be zero, as long as is it bounded. If on the other hand, debt does to infinity as T goes to infinity, it will compete with the $\lim_{T\to\infty} \left(\frac{1}{1+r}\right)^{T-1}$ going to zero, and the transversality condition will not be fulfilled.

consider that honoring its debt is too costly, and hence will default. Then, a country will be violating its transversality condition if previous trade account (or current account) imbalances have led to a current level of net foreign borrowing, $\bar{B}_t$, such that:

$$\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^{i-1}\bar{C} > \sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^{i-1}\bar{Y} - \bar{B}_t \tag{25}$$

$$\Rightarrow \frac{1}{1-\frac{1}{1+r}}\bar{C} > \frac{1}{1-\frac{1}{1+r}}\bar{Y} - \bar{B}_t$$

$$\Rightarrow \frac{1+r}{r}\bar{C} > \frac{1+r}{r}\bar{Y} - \bar{B}_t$$

where we have made use of the formula for an infinite geometric sum. From the above, we can compute a threshold level of debt to GDP ratio above which, the economy will be defaulting at some point in time, by changing the inequality to an equality and substituting $\bar{B}_t$ with $\bar{B}$:

$$\Rightarrow \frac{\bar{B}}{\bar{Y}} = \frac{1+r}{r}\left(\frac{\bar{Y}-C}{\bar{Y}}\right) \tag{26}$$

If the current net debt level is at or above $\frac{\bar{B}}{\bar{Y}}$ as defined in (26), then at some point in time, it must be that consumption has to go below $\bar{C}$, which we know (per definition here) would trigger a default on the foreign debt of the country. Obviously, there are a lot of unreasonable assumptions here, and most - if not all - of the information we need to make this evaluation is not known a apriori. This is why we look at a related, but different, approach to sustainability below, which starts from the notion that there is a maximum level of sustainable net foreign liabilities, and then makes a guesstimate about this level, often based on historical levels of net foreign liabilities.

2.2 Current Account Sustainability and Stabilization of the Net Foreign Debt

The definition of sustainability of the level of net foreign debt above gives us an idea of what it means, but it is not very applicable when it comes to actually evaluating whether a given current account position is sustainable or not, partly because we do not know future income and interest rates, but particularly because we do not know this threshold level of C. A more applicable and often used measure of current account sustainability takes as a starting point some necessarily ad hoc choice of a level of net foreign debt, above which the economy in question will be in danger of not being able to honor its debt obligations. With such a threshold in mind, a given current account position can then be evaluated as to whether a sustained current account position of that magnitude will eventually lead to the economy surpassing that threshold level or not.

To formalize this approach, call the current account deficit in percent of GDP for $\frac{CA_t}{Y_t}$, and net foreign liabilities in percent of GDP for $\frac{B_t}{Y_t}$. Call the threshold level of the net foreign liabilities, beyond which the country is likely to enter into default, for x. Assume that the economy has a constant real GDP growth rate of g per period. In this case, the net foreign liabilities accumulation equation is given by

$$\begin{aligned}
\frac{B_{t+1}}{Y_{t+1}} &= \left[\frac{CA_t}{Y_t} + \frac{B_t}{Y_t}\right]\frac{Y_t}{Y_{t+1}} \\
&= \left[\frac{CA_t}{Y_t} + \frac{B_t}{Y_t}\right]\frac{(1-g)Y_{t+1}}{Y_{t+1}} \\
&= (1-g)\left[\frac{CA_t}{Y_t} + \frac{B_t}{Y_t}\right]
\end{aligned}$$

(Note that we do not assume anything about the interest rate paid on the net foreign liabilities, since these interest payments are included in the current account deficit. We also ignore any potential capital gains or losses, including the effect of changes in exchange rates on the value of net foreign liabilities, on the net foreign liabilities which would not be included in the current account)

According to the formula, in order to keep the net foreign liabilities as a percentage of GDP constant in time, i.e. $\frac{B_{t+1}}{Y_{t+1}} = \frac{B_t}{Y_t}$, we would need a certain level of the current account deficit in percent of GDP, call this level z:

$$\begin{aligned}
\frac{B_t}{Y_t} &= (1-g)\left[z + \frac{B_t}{Y_t}\right] \\
&\Rightarrow \frac{B_t}{Y_t} - (1-g)\frac{B_t}{Y_t} = (1-g)z \\
&\Rightarrow \frac{B_t}{Y_t}\frac{(1-(1-g))}{(1-g)} = z \\
&\Rightarrow \frac{B_t}{Y_t}\frac{g}{1-g} = z \qquad (27)
\end{aligned}$$

(27) gives us our formula for evaluating the sustainability of a given current account position. Take a few numerical examples based more or less on the current numbers for the US.

First, if we want a net foreign asset ratio to GDP to be stabilized at 25 percent, and we have a real long term growth rate of 3 percent, we would get

$$z = 0.25 \cdot \frac{0.03}{1-0.03} = 0.007732 \approx 0.8\%$$

That is, the current account deficit in percent of GDP would have to stay at or below 0.8 percent on the average for it to be sustainable. A much higher current account deficit would imply that a correction would have to happen in the not too distant future to stay below the 25 percent net foreign liabilities threshold. Conversely, if we have a current account deficit ratio to GDP of 5 percent, and a GDP growth rate of 3 percent-, the net foreign liabilities ratio would eventually stabilize at

$$\frac{B_t}{Y_t} = \frac{1-0.03}{0.03} \cdot 0.05 = 1.617 \approx 160\%$$

Finally, if we believe that the limit to sustainability of the net foreign liabilities ratio occurs at 100 percent, and we assume a long term growth rate of 3 percent, the associated maximum sustainable average current account de…cit is

$$z = 1 \cdot \frac{0.03}{1 - 0.03} = 0.0309 \approx 3\%$$

The problem with this simple accounting approach is of course that no one really knows at what level the burden of net foreign liabilities becomes too hard to bear for a country. All it tells us is that if a real current account de…cit exceeds 3 percent in a country with an average long term real growth rate of 3 percent, and that if you believe that 100 percent net foreign liabilities to GDP ratio is the highest sustainable level, then the current account de…cit is not sustainable for a longer time period and will have to reverse at some point. It does not say anything about how long a time period the excess current account de…cit can go on and when and how we will see a reversal, nor whether the limit of 100 percent is really appropriate.

Note that we can apply exactly the same method for evaluating whether a …scal de…cit is sustainable or not, and whether it will eventually lead to a higher or lower debt to GDP ratio than currently observed.

3 Government De…cits and the Current Account

3.1 Ricardian Equivalence...

Under full Ricardian equivalence, a government de…cit due to tax cuts should have no e¤ect on overall savings of the economy. An increase in the government de…cit by a reduction in taxes and an increase in debt …nancing will always be entirely matched by a one to one increase in private savings. Current private consumption will be left unchanged. This is because households know that a tax cut today will be matched by an increase in taxes tomorrow by the entire amount plus interest, in order to repay the debt plus interest incurred by the tax cut. Households will therefore place the entire tax cut in interest bearing bonds.

The simple model of the intertemporal approach to the current account that we treated in Section (1) features Ricardian equivalence, and therefore, any budget de…cit will be met by equally increased savings by the private sector, leaving the total external de…cit unchanged. To see this, assume that we have a two period endowment economy (i.e. output is exogenous and there is no investment - this assumption is not essential for the results, but it simpli…es the treatment). Recall that the government intertemporal budget constraint is given by

$$\tau_t + \frac{\tau_{t+1}}{1+r} = G_t + \frac{G_{t+1}}{1+r}$$

Suppose that the government, from a situation of budget balance each period, i.e. $\tau_i = G_i$, $i = t; t+1$, decides to give a debt …nanced tax relief of $\Delta\tau_t < 0$ in the first period without changing spending in the two periods. This implies a fall in government savings by the same amount, namely $\Delta S_t^G = \Delta\tau_t$. Obviously, for the intertemporal budget constraint to hold, this implies that in the second period, taxes must be increased so as to repay the debt induced by the first period tax cut. Specifically, taxes in period two must increase by $-(1+r)\Delta\tau_t$. Now, how does this first period tax cut affect the household net present value of lifetime disposable income? The answer is that it does not. Per definition, we know that

$$\Delta x_t = -\Delta\tau_t - \frac{\Delta\tau_{t+1}}{1+r} = -\Delta\tau_t - \frac{-(1+r)\Delta\tau_t}{1+r} = 0.$$

Since the time profile for the income for the household does not matter for the spending profile when the household has unlimited access to credit and lending at the international interest rate, r, the household will not change spending at all in response to this tax cut. The entire tax cut will be saved and used for paying the higher taxes in the subsequent period. Hence, the change in private savings due to the tax cut is $\Delta S_t^P = -\Delta\tau_t$. Total savings of the economy are left unchanged, and hence, the current account is left unchanged. The higher public deficit is not translated into a higher external deficit. Whether or not we have Ricardian equivalence is therefore relevant in the present context because under Ricardian equivalence, a debt financed tax cut does not affect the current account at all, and the hypothesis that government deficit cause current account deficits (the Twin Deficit hypothesis) does not hold.

Ricardian equivalence does not ensure the neutrality of all types of government deficits however. If the deficit is increased by a debt financed increase in spending, the private sector will react. This is because a change in spending will affect the net present value of disposable lifetime income of the private agent. In our model, this situation could be characterized by a current debt financed spending increase of $\Delta G_t > 0$, holding spending in the second period and taxes in the current period constant, $\Delta\tau_t = \Delta G_{t+1} = 0$. According to the government budget constraint, this would imply a tax increase in the second period of $\Delta\tau_{t+1} = -(1+r)\Delta G_t$. Government savings would hence fall in the first period by

$$\Delta S_t^G = -\Delta G_t;$$

and increase in the second period by

$$\Delta S_{t+1}^G = (1+r)\Delta G_t - r\Delta G_t = \Delta G_t;$$

where the subtraction of interest payments come from the fact that these are the interest payments on the increased public debt acquired in the first period in order to finance the higher spending. This policy measure would change the net present value of income of the household exactly by the spending increase:

$$\Delta x_t = -\Delta\tau_t - \frac{\Delta\tau_{t+1}}{1+r} = 0 - \frac{-(1+r)\Delta G_t}{1+r} = \Delta G_t.$$

To see how this affects household spending and savings decisions, simplify by assuming that $\beta = (1+r)$, so that the Euler equation will dictate an optimal allocation of spending across time given by identical levels of spending in the two periods. We therefore know that any change in income of the household will be distributed equally across the two periods, such that

$$\Delta C_t = \Delta C_{t+1}.$$

From the household budget constraint, we have that that the lower lifetime net income is distributed according to

$$-\Delta G_t = \Delta C_t + \frac{\Delta C_{t+1}}{1+r}.$$

Solving these two equations for ΔC_t and ΔC_{t+1}, we get:

$$\Delta C_t = \Delta C_{t+1} = -\frac{1+r}{2+r}\Delta G_t.$$

Since nothing happens to the period income in the first period, household savings in the first period will change with the reduction in spending, so that we have

$$\Delta S_t^P = \frac{1+r}{2+r}\Delta G_t = -\Delta B_t$$

while in the second period, the higher taxes are levied but the household will receive interest payments from its purchase of bonds for the higher savings in the first period, leading to a private savings change of

$$\begin{aligned}
\Delta S_{t+1}^P &= -(1+r)\Delta G_t + \frac{1+r}{2+r}\Delta G_t - r\Delta B_t \\
&= -(1+r)\Delta G_t + \frac{1+r}{2+r}\Delta G_t - r\frac{1+r}{2+r}\Delta G_t \\
&= -\frac{1+r}{2+r}\Delta G_t
\end{aligned}$$

Adding private and public savings changes gives us the total effect of the policy change on the current account:

$$\begin{aligned}
\Delta CA_t &= -\Delta S_t = \Delta G_t - \frac{1+r}{2+r}\Delta G_t = \frac{1}{2+r}\Delta G_t \\
\Delta CA_t &= -\Delta S_t = -\Delta G_t + \frac{1+r}{2+r}\Delta G_t = -\frac{1}{2+r}\Delta G_t
\end{aligned}$$

The policy change will thus lead to an increase in the current account deficit in the first period by less than the total amount of government spending increase, since households neutralize some of the spending increase by smoothing its impact in the second period to the first period. In the second period, we will see a reduction in the current account deficit, as households reduce their spending the the government runs a surplus with the higher tax revenues. In this case, private sector reactions to the change in the deficit make the impact on the current account smaller and smoothed across the two periods, but does not neutralize it. This example thus shows that the reason for a change in the de…cit matters for the e¤ect of the de…cit on the private sector behavior and on the current account under Ricardian equivalence.

Relaxing the assumption of Ricardian equivalence, fully or just partially, is for example necessary to arrive at the hypothesis of Twin De…cits, stating that a government de…cit feeds directly into the current account, leading to a current account de…cit, all else equal.

Ricardian equivalence requires very strict simplifying assumptions for it to be a feature of a model, and these assumptions are unlikely to be met fully in reality. A few examples follow.

3.2 ... And When it Breaks Down

Ricardian equivalence requires, among other assumptions, that private agents have unlimited access to capital markets and hence consumption smoothing; that they maximize over the same horizon as the government (i.e. private agents live in both periods in the example above), that taxes are not distortionary, that private and public interest rates are the same, and that consumers are not myopic, among many other things. If these conditions are not met, we might see a partial or full breakdown of Ricardian equivalence. Take the two …rst exceptions as examples, each in turn.

3.2.1 Example 1: Credit constrained households

First, take the example of a model in which private agents - households following the example above - do not have unlimited access to capital markets. In the extreme, we can assume that households have NO access to capital markets, just to make the point clear, but it is in fact enough to assume that households only have partial access to capital markets for Ricardian equivalence to break down[6]. Assume however, that the government does have access to international capital markets for lending and borrowing, and look at an endowment economy without investment. In this example, the only savings and borrowing of the economy would be public. Households would spend exactly their net income each period

$$C_t = Y_t \text{ ¡ } ¿_t \tag{28}$$

and

$$S_t^p = Y_t \text{ ¡ } ¿_t \text{ ¡ } C_t = 0$$

for all t. The government can run de…cits or surpluses, and holds $B_{t \text{ ¡ } 1}$ amount of debt from the previous period on which is must pay interest (the debt of the

[6] For example, when assumed that there are two types of households in the economy, one type which has access to capital markets and another type which does not, we have limited capital market access. The later type of households are currently referred to as "Rule of Thumb" households. See for example Gali, Lopez-Salido and Valles, 2004, Understanding the E¤ects of Government Spending on Consumption, ECB Working Paper no. 339.

previous period was assumed zero in the model of section one, but we leave it in here this time):

$$S_t^g = \tau_t - G_t - rB_{t-1}:$$

Hence, total savings of the economy would become

$$S_t = S_t^g + S_t^p = \tau_t - G_t - rB_{t-1}$$

Now look at the definition of the current account, which is equal to total production/income less total spending in the economy (net exports) less net interest payments on foreign debt[7]:

$$CA_t = Y_t - C_t - G_t - rB_{t-1}$$

Inserting the equation for household spending, (28), into the above yields

$$CA_t = \tau_t - G_t - rB_{t-1}$$

Thus, the current account definition in the case of credit constrained households is equal to the government budget deficit! If the government runs a balanced budget, i.e. $S_t^g = 0$, total spending of the economy would be equal to production and income, there would be zero net savings, and hence current account balance (but note that net exports could be positive, if the government has debt to pay interests on). Conversely, from a situation of government budget balance, a tax-cut one period, keeping government spending constant, would lead to a budget deficit and in turn a higher net income for the households. This in turn would lead to a one for one increase in household spending in that period, and a one for one deterioration the current account. This situation, in which we have a government deficit directly causing a current account deficit, is known as the twin deficit hypothesis.

3.2.2 Example 2: Overlapping Generations

Now take the second example of a breakdown in Ricardian equivalence, and assume that households do not live to the end of time (i.e. to the last period of the model). Take an economy with an infinite horizon, and assume that individual households only exist in two periods, one in which household members are young, work, and save for retirement, and one in which household members are old and retired, living off savings, as in a simple overlapping generations model. Young and old households coexist in each period, and in each period, an old household dies and a young household is born. Consider now a tax cut inducing a government deficit, which in turn is financed by taking up government debt to be repaid over a certain number of periods. The young households living at the time of the tax cut suddenly have a higher net income in that period, but will also expect to pay slightly higher taxes the next period, when some of

[7] Note that contrary to the usual definition, we have a minus sign before interest payments. This is because we assume that B_t is net foreign liabilites, not net foreign assets, as is traditionally the case.

the government debt is to be repaid. But they will not be around to pay o¤ the entire government debt, and therefore, their net income is increased by a little less than the full tax cut. They smooth their higher …rst period income over their two-period lifetimes, and therefore spend more in both periods. The e¤ect on old households' net income is even more pronounced, since they will not be around to pay the higher future taxes associated with the debt increase at all. They will therefore spend the entire amount of the tax cut in the same period. In fact, most of the tax bill for the current de…cit will be passed on to future generations, who will hence be consuming less. The sum of all these actions is that total savings of the economy will fall in the period of the debt …nanced tax cut, and in turn, the current account de…cit will deteriorate. The current account deterioration is not one to one with the budget de…cit, since a portion of the tax cut after all is internalized by the young households who will be there to pay some taxes in the period after the tax cut, and who will want to save some of their tax cut for consumption tomorrow. Due to the increased tax burden on future generations, the long term e¤ect of a tax cut is lower future spending by households and in turn current account surpluses.

Hence, in a simple overlapping generations model, the government budget de…cit leads to a short term current account de…cit, a medium term smaller current account de…cit, and a long term small current account surplus starting from the period after the tax cut.

4 References

Obstfeld, Maurice and Kenneth Rogo¤ (O&R), 1996, Foundations of International Macroeconomics, Massachusetts Institute of Technology.

Figure 1. The Intertemporal Production Possibilities Frontier

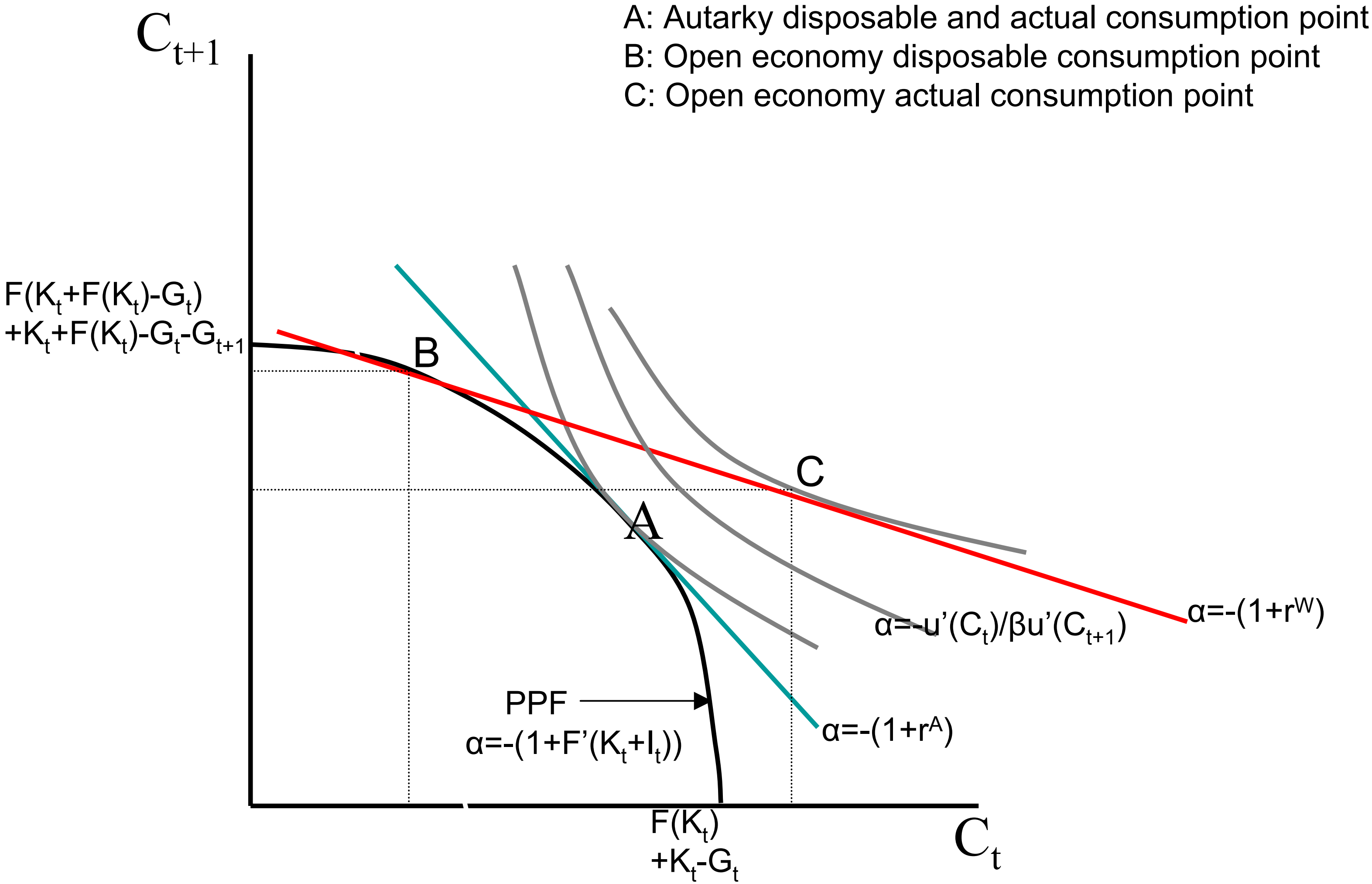

Figure 2. Example: An increase in expected productivity in the small country